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ON THE KINEMATIC PROBABILITY OF TERMINAL

BALLISTICS WITH INITIAL DISPERSION

By

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U. S. ARMY WEAPONS COMMAND
ROCK ISLAND, ILLINOIS

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ABSTRACT

The terminal ballistic dispersion of a non-rotational, small-caliber weapon system is obtained on a priori knowledge of the initial dispersion induced by the weapon system proper.

The study considers a dynamic system whose behavior is governed by a system of differential equations having probabilistic initial conditions. The behavior of the system, in terms of the kinematic probability, is then determined as a function of time and spatial variables.

Subsequently, in contrast to the customary method in the evaluation of weapon system effectiveness, a measure of effectiveness - probability of hit - is obtained as a function of initial dispersion. Also, numerical examples as well as discussion of the results are given.

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I. INTRODUCTION

Conventionally, the terminal effect of projectiles, related to a weapon system, is analyzed based on a priori knowledge of the dispersion at a target. The probability density function of the dispersion is specified and its parameters are given. The probability of hit and other effectiveness measures are then sought [1,2].

Such an approach is pertinent if we address ourselves strictly to the problem of effectiveness at the target only, and the terminal dispersion taken is well substantiated experimentally. When the projectiles are related to a new conceptual weapon system for which no physical experiments have yet been performed, a question naturally arises about the soundness of the above approach.

To a larger extent than this consideration on the target alone, if we are more concerned with the effectiveness of a conceptual weapon system which incurs certain terminal effects on a given target, hoping eventually to bolster the rationale for preliminary engineering design of the system, it is hardly plausible that we could merely consider the terminal dispersion as a priori knowledge. What appears to be needed in the treatment of such a problem is a terminal dispersion obtained on the base of some criteria pertaining to the weapon system proper.

We can schematically envision three regions of concern: conceptual weapon system, exterior ballistics and terminal effects, as shown in Fig. 1. The region D which is the intersection of the regions A and B contains

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information relating to the weapon system, whereas the region E which is the intersection of the regions B and C has information pertaining to the terminal dispersion. It is clear that information arises from D, coupled with B, would affect information about E and subsequently about C. The specific point of concern now is what particular information from D must we generate and how should it arrive at E?

Since a measurable information at E is the terminal dispersion, it is reasonable to consider its counterpart at D - the initial dispersion. For this reason, the a priori knowledge on the initial dispersion of a conceptual weapon system is necessary, for it is formulated on the base of our understanding of the intrinsic properties such as weapon dynamic parameters [3] of the system under consideration. The terminal dispersion is then rigorously sought analytically

In this report the probability distribution of terminal dispersion and subsequently some terminal effects are obtained for a given target and a given probability of initial dispersion. The problem is considered as a stochastic dynamic process for a time duration ranging from t_0 to t. The initial probabilities of the kinematic, or state, variables - displacement and velocity - are given a priori.

As of special interest, an exterior ballistic model for small mass is considered. It is shown that a linearization of the system dynamic equation can be realized. A method of attack rendering the transformation of the kinematic probabilities from one time to another is described. Solutions of the problem are then presented. A numerical example is given, and the results are discussed.

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II. STATEMENT OF THE PROBLEM

Let us consider a weapon system of caliber d delivering a small projectile of mass m into a target. The projectile has a muzzle velocity, V_m, with respect to a given rectangular Cartesian coordinate system, x-y-z, which is also an inertia frame, having the y coordinate axis in the range direction. The range to the target is short such that the air density ρ and the temperature in the neighborhood of the trajectory, as well as the gravitational acceleration, g, can be taken as constant. It follows the velocity of sound a_0 is also constant.

Also let us assume the air is almost still and the projectile moves with its axis tangent to its trajectory. The only force acting on the projectile are the drag and the gravitational force. We can now write down the normal equations of motion for the projectile as follows [4]

$$\ddot{x} = -K_D \rho d^2 \, V \, V_x/m$$

$$\ddot{y} = -K_D \rho d^2 \, V \, V_y/m \qquad (2.1)$$

$$\ddot{z} = -K_D \rho d^2 \, V \, V_z/m - g$$

where V_x, V_y and V_z are the components of V along x,y and z coordinate axes respectively, and K_D is the drag coefficient.

It has been shown in [5,6,7] that K_D can be considered as a function of Mach number for (2.1) and for certain range of Mach numbers K_D is inversely proportional to the Mach number of the projectile. Consequently, it can be written as follows

$$K_D = \frac{\beta a_0}{V} \qquad (2.2)$$

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where β is a positive constant of proportionality and depends on the type of projectile, in particular, the geometrical configuration.

A simple linearization of (2.1) can be done by substituting (2.2) into (2.1).

$$\ddot{x} = \gamma\dot{x}$$
$$\ddot{y} = \gamma\dot{y} \qquad\qquad (2.3)$$
$$\ddot{z} = \gamma\dot{z} - g$$

where $\gamma = -\beta a_0 \rho \, d^2/m$.

Letting $x_1 = x, x_2 = y$ and $x_3 = z$, we can reduce (2.3) to a system of first order linear system, with the initial conditions at $t_0 = 0$ specified, as follows:

$$\dot{x}_i = x_{i+3} \qquad\qquad i=1,2,3$$
$$\dot{x}_j = \gamma x_j \qquad\qquad j=4,5 \qquad\qquad (2.4)$$
$$\dot{x}_6 = \gamma x_6 - g$$

and $\quad x_i(0) = x_i^0 \qquad\qquad i=1,2,\cdots,6$

Now, x_i^0 is not deterministic for there exists initial dispersion. Therefore, (2.4) is a system of stochastic differential equations with stochastic initial conditions.

Our problem can now be stated succinctly as follows: given the initial joint probability density function of x_i^0, find the joint and marginal probability density functions of $x_i(t)$ for (2.4).

A natural follow-up question is as follows: given a target T, find the probability of $x_i(t)$ over T, which is the probability of hit.

In this report, we presume that x^0,, a random vector with components x_i^0, has normal joint probability density function with independent marginal probability density functions characterized by two parameters - mean μ_i^0 and variance $(\sigma_i^0)^2$. A rectangular parallelepiped target, as shown in Figure 2, is considered, i.e.

$$T = \{(x_1,x_2,x_3)\epsilon E^3 | s_1 \leq x_1 \leq s_2, \ s_3 \leq x_2 \leq s_4, \ s_5 \leq x_3 \leq s_6\} \quad (2.5)$$

where s_i is specified.

III. METHOD OF SOLUTION

For convenience let us consider, in general, a system of n first order differential equations written in matrix form

$$\dot{y} = f(y,t) \qquad\qquad t > 0$$
$$y(0) = y^0$$

(3.1)

where

$$y = (y_1, y_2, \cdots, y_n)^T$$

$$f = (f_1, f_2, \cdots, f_n)^T$$

and

$$y(0) = (y_1^0, y_2^0, \cdots, y_n^0)^T.$$

Geometrically (3.1) describes the dynamics of a point in E^n with time t as a parameter. If the initial condition is known only probabilistically, then the solution of the vector equation (3.1) is a random vector even though the equation itself is deterministic. Thus, we can solve for the solution y, a random vector, by considering these equations as if they were deterministic ones, and obtain the solution in the form

$$y = \phi(y^0, t)$$

(3.2)

We observe that in (3.2) ϕ is a deterministic transformation mapping the random vector y^0 defined on a probability space Ω into a new random vector y defined on the same space as shown in Figure 2A. Moreover, this mapping is deterministic. Hence, ϕ is probability – preserving i.e.

$$P(y \in \phi A) = P(y^0 \in A)$$

(3.3)

where P denotes a probability distribution and A is a measurable subset of the reals.

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These notions lead us readily to an approach for solving (2.4) based on the previous works [8,9]. For convenience, let us recapitulate the results in a form of a theorem without proof:

Theorem

Given the system (3.1) and the initial joint probability density function $p(y^0)$ at $t = t_0$. If for all $i,j = 1,2, \cdots ,n$

$$| \frac{\partial f_i}{\partial y_j} | \leq M \tag{3.4}$$

in the domain of definition, where M is a positive constant. Then, the probability density function for $t > t_0$, denoted by $p(y,t)$, which satisfies the following partial differential equation

$$\frac{\partial p}{\partial t} + \sum_{i=1}^{n} \frac{\partial(pf_i)}{\partial y_i} = 0 , \tag{3.5}$$

is given by the formula

$$p(y,t) = p(y^0) | \frac{\partial y^0}{\partial y} | \tag{3.6}$$

where $\frac{\partial y^0}{\partial y}$ is the Jacobian of the initial vector y^0 with respect to the vector y.

Note that it is essential for this approach that the inverse of y in (3.2), i.e.,

$$y_0 = \Phi(y) \tag{3.7}$$

where $\Phi = \phi^{-1}$,

must be attainable in order to utilize (3.6).

IV. SOLUTION OF THE PROBLEM

We observe that the theorem is applicable to (2.4). Hence, our
problem reduces to obtain the inverse of the solution of (2.4).

Let us denote

$$x^0 = (x_1^0, x_2^0, \cdots, x_6^0)^T \tag{4.1}$$

and

$$x = (x_1, x_2, \cdots, x_6)^T. $$

Solving (2.4) for x^0, we obtain

$$x^0 = Ax+B \tag{4.2}$$

where

$$A = \begin{bmatrix} 1 & 0 & 0 & a & 0 & 0 \\ 0 & 1 & 0 & 0 & a & 0 \\ 0 & 0 & 1 & 0 & 0 & a \\ 0 & 0 & 0 & b & 0 & 0 \\ 0 & 0 & 0 & 0 & b & 0 \\ 0 & 0 & 0 & 0 & 0 & b \end{bmatrix} \tag{4.3}$$

$$B = (0,\ 0,\ \frac{-g(a+t)}{\gamma},\ 0,\ 0,\ -ag)^T \tag{4.4}$$

$$a = (b-1)/\gamma \tag{4.5}$$

and

$$b = e^{-\gamma t} \tag{4.6}$$

The absolute value of the Jacobian of (4.2) is then

$$\left| \frac{\partial x^0}{\partial x} \right| = e^{-3\gamma t} \tag{4.7}$$

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Now, the joint probability density function of x at any time t can be written by using (4.2) with the normal joint probability density function of x^0 as follows:

$$p(x,t) = \frac{e^{-3\gamma t}}{(2\pi)^3 \prod\limits_{i=1}^{6} \sigma_i^0} \exp\left\{-\frac{1}{2} \sum\limits_{i=1}^{6} \left(\frac{x_i - \nu_i}{\sigma_i^0}\right)^2\right\} \tag{4.8}$$

where

$$\nu_i = \mu_i^0 \qquad\qquad i=1,2,4,5$$

$$\nu_3 = \mu_3^0 + \frac{g}{\gamma}(t+a)$$

and

$$\nu_6 = \mu_6^0 + ag.$$

We have tacitly used the independence among the marginal probability densities of x^0 as stated.

It follows that the marginal probability density of x is expressible as

$$p(x_i,t) = \int_{E^{n-1}} p(x,t)\,d\overline{x} \tag{4.9}$$

where

$$d\overline{x} = \prod\limits_{\substack{j=1 \\ j\neq i}}^{n} dx_j \qquad\qquad \text{for } n=6, \text{ and}$$

the integral sign denotes n-1 integrals over E^{n-1}. Thus we can write with the use of (4.8) and (4.9) following results

$$p(x_i,t) = \frac{b}{\sqrt{2\pi}\beta_i} \exp\left\{-\frac{1}{2}\left[\frac{x_i - (\nu_i - a\,\nu_{i+3})}{\sigma_i^0}\right]^2\right\} \tag{4.10}$$

where

$$\beta_i = (a\,\sigma_{i+3}^0)^2 + (b\,\sigma_i^0)^2 \qquad\qquad i=1,2,3$$

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and

$$p(x_i, t) = \frac{1}{\sqrt{2\pi} \ \sigma_i^0} \exp \left[-\frac{1}{2} \left(\frac{bx_i - v_i}{\sigma_i^0} \right)^2 \right] \qquad i=4,5,6 \qquad (4.11)$$

The mean of x_i is expressible by definition as

$$\mu_i(t) = \int_{-\infty}^{\infty} x_i \ p(x_i, t) dx_i \qquad (4.12)$$

which yields the following results with the use of (4.10) and (4.11):

$$\mu_i(t) = v_i - av_{i+3}/b, \qquad i=1,2,3 \qquad (4.13)$$

$$\mu_i(t) = v_i/b \qquad i=4,5,6 \qquad (4.14)$$

Similarly, the mean square value of x_i can be written as

$$E[x_i^2(t)] = \int_{-\infty}^{\infty} x_i^2 \ p(x_i, t) dx_i \qquad (4.15)$$

which gives

$$E[x_i^2(t)] = \frac{\beta_i}{b^2} + \left(v_i - \frac{av_{i+3}}{b} \right)^2, \qquad i=1,2,3 \qquad (4.16)$$

$$E[x_i^2(t)] = (\sigma_i^2 + v_i^2)/b^2 \qquad i=4,5,6 \qquad (4.17)$$

Since the variance of x_i is related to its mean and mean square value as

$$\sigma_i^2(t) = E[x_i^2(t)] - \mu_i^2(t) \qquad (4.18)$$

the following results are immediate

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$$\sigma_i^2(t) = \beta_i/b^2 \qquad\qquad i=1,2,3 \qquad\qquad (4.19)$$

$$\sigma_i^2(t) = [\sigma_i^0/b]^2 \qquad\qquad i=4,5,6 \qquad\qquad (4.20)$$

For the target T of (2.5) the probability of hit at a given time t, $P_h(t)$, can be obtained by first finding the joint probability density function of x_i, $i=1,2,3$; i.e.

$$p(x_1,x_2,x_3,t) = \int_{E^3} p(x,t)d\overline{x} \qquad\qquad (4.21)$$

where

$$d\overline{x} = dx_4 \; dx_5 \; dx_6$$

Subsequently, $P_h(t)$ over T can be written as follows

$$P_h(t) = \int_T p(x_1,x_2,x_3,t)d\overline{x} \qquad\qquad (4.22)$$

where $d\overline{x} = dx_1 \; dx_2 \; dx_3$ and the integral sign denotes a triple integral over T.

Substantially, by utilizing the results of (4.21) into (4.22) and employing the notion of error functions, we obtain the following expression:

$$P_h(t) = \prod_{i=1}^{3} [\mathrm{erf}(\eta_{2i}) - \mathrm{erf}(\eta_{2i-1})]/2 \qquad\qquad (4.23)$$

where

$$\eta_{2i-1} = b(s_{2i-1} - \mu_i)/\sqrt{2\beta_i}$$

$$\eta_{2i} = b(s_{2i} - \mu_i)/\sqrt{2\beta_i} \qquad .$$

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V. NUMERICAL EXAMPLE

As an illustration, let us consider the following weapon system in the milieu as given:

$$m = 3.774 \times 10^{-4} \qquad \text{slug (85 grain)}$$

$$d = 1.9685 \times 10^{-2} \qquad \text{ft} \quad (6.0\text{mm})$$

$$\beta = 0.25$$

$$a_0 = 1120.27 \qquad \text{ft/sec}$$

$$\rho = 2.377 \times 10^{-3} \qquad \text{slug/ft}^3$$

$$g = 32.174 \qquad \text{ft/sec}^2$$

The system gives initial dispersion with means (ft) and standard deviations (ft) as follows:

$$\mu_1^0 = 3. \times 10^{-5} \qquad\qquad \sigma_1^0 = 1.5 \times 10^{-5}$$

$$\mu_2^0 = -6 \times 10^{-5} \qquad\qquad \sigma_2^0 = 3. \times 10^{-5}$$

$$\mu_3^0 = 2 \times 10^{-5} \qquad\qquad \sigma_3^0 = 10^{-5}$$

$$\mu_4^0 = 2.0 \ (\text{ft/sec}) \qquad\qquad \sigma_4^0 = 0.1 \ (\text{ft/sec})$$

$$\mu_5^0 = 3500.0 \ (\text{ft/sec}) \qquad\qquad \sigma_5^0 = 50.0 \ (\text{ft/sec})$$

$$\mu_6^0 = 3.0 \ (\text{ft/sec}) \qquad\qquad \sigma_6^0 = 0.2 \ (\text{ft/sec})$$

A rectangular parallelepiped target T has the following dimensions (ft) for its s_1:

$$s_1 = -3.0 \qquad\qquad s_2 = 3.0$$

$$s_3 = -2.0 \qquad\qquad s_4 = 2.0$$

$$s_5 = -3.5 \qquad\qquad s_6 = 3.5$$

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Let us consider that T is zeroed in, and is situated at a range x_2^i.
Since T is a volume target, let us specifically address ourselves to a
particular time t^i which is related to x_2^i with nil initial variance.
A deterministic relation can be readily obtained for t^i from (2.4) as
follows:

$$t^i = \frac{1}{\gamma} \ln[1 + \gamma(x_2^i - x_2^0)/x_5^0] \qquad (5.1)$$

As an illustration, a set of ranges up to x_2^i = 1000 meters is con-
sidered. For any value of x_2^i in this set, (5.1) yields the corresponding
t^i, and the results in Section 4 are then subsequently utilized in seeking
numerical solutions.

In Fig. 3 the "flight time" t^i across the range x_2' of interest is
given. The rangeward mean velocity μ_5 is shown in Fig. 4. In Fig. 5
the mean lateral displacement and velocity μ_j and μ_4 are shown. The
rangeward mean transverse displacement and velocity, μ_3 and μ_0 are given
in Fig. 6.

Fig. 7, 8 and 9 depict the rangeward standard deviations, σ_1 and
σ_{1+3}, respectively. The maxima of the marginal density function $p(x_1)$
are given in Fig. 10, 11 and 12. Finally, the effects of target dimensions
and initial dispersion on the probability of hit $P_h(t^i)$ are shown,
respectively, in Fig. 13 and 14.

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VI. DISCUSSION AND CONCLUSIONS

A solution to the problem of finding the kinematic probability of terminal ballistics, given an initial dispersion, has been presented in this report. The feasibility of such a solution implies that the initial dispersion of a weapon system, rather than the terminal one, should be addressed rigorously in a weapon system effectiveness study.

For any given range and target configuration, the terminal effectiveness, such as probability of hit, is simply a consequential result once the initial dispersion of the weapon proper is attained. In contriving the preliminary design layout of a conceptual weapon system, the initial dispersion is therefore one of the most significant factors for considerations. It is a key juncture to the overall system effectiveness.

The method of attack presented here is general to the extent that no constraint is posed on the probability of initial dispersion. Indeed, the methods can also be employed for the cases other than normal distribution as specified for the problem. The theorem is applicable to a nonlinear system as well as a linear one whenever (3.7) is obtainable.

As we can observe from (4.2) and (4.3), X_i hence X_i^0 are coupled among each other. The common concern for an initial dispersion in terms of the initial displacement alone is therefore not adequate. Clearly, we should also take the initial velocity due to weapon dynamics in addition to the muzzle velocity, into account for consideration of an initial dispersion.

The result of (4.23) for a volume target can readily be applied to an area target of rectangular shape by collapsing one dimension and letting one-half of the difference of the error functions for that coordinate

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assume the value of unity in computation. Moreover, (4.23) indicates that $P_h(t)$ for a volume target is bounded by one-half of the corresponding value for the area target.

Using (4.14) the absolute value of the mean striking velocity $\bar{\nu}_v(t)$ can be written as

$$\bar{\mu}_v(t) = [\sum_{i=4}^{6} \nu_i]^{1/2}/b \tag{6.1}$$

We observe as $\mu_v(t)$ vanishes as t increases to infinity. On the other hand, from (4.13) we can write the asymptotic value for the mean displacement $\mu_i(t)$, $i=1,2,3$ as follows

$$\mu_i(t) \rightarrow \nu_i + \frac{\nu_{i+3}}{|\gamma|} \tag{6.2}$$

which provides an upper bound for terminal mean displacement. The parameter γ in (2.4) and (4.6) can lead us to consider $1/|\gamma|$ as the time constant of the system.

The direction cosines α_i of the striking velocity vector are obtainable from the ratio of (4.14) and (6.1) as follows:

$$\cos \alpha_i = \frac{\nu_i}{[\sum_{j=4}^{6} \nu_j^2]^{1/2}} \qquad i=4,5,6 \tag{6.3}$$

We observe that α_i is time invariant; i.e., the α_i at the terminal point coincides with that of any departure point.

It should be noted that the modeling does not involve rotational effect of a projectile. Furthermore, the relation (2.2) should be observed.

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The advantage in choosing K_D in the form (2.2) is two-fold. It not only reduces the nonlinear systems of (2.1) into a linear system but also gives a "better" approximation to the empirical curves of K_D for the ranges of our concern (around 1 to 5 Machs), in contrast to the customary assumption being constant.

The numerical results in Fig. 4 through 6 can be obtained identically by solving (2.4) as a deterministic system with $x_i^0 = \mu_i^0$. The standard deviations shown in Fig. 7 to 9 are almost linear with respect to the range. The maximum of displacement probability density function, $p(x_i)_{max}$, $i=1,2,3$, decays rapidly before 400m as shown in Fig. 10, 11 and 12. In the same figures, the maximum of velocity probability density function is shown having slow diversion initially up to a point about half way of the range, and then an escalation.

In Fig. 13, it is shown that as the target dimension increases the probability of hit $P_h(t^i)$ increases as well. When the initial dispersion decreases, $P_h(t^i)$ increases as shown in Fig. 14. The sensitivities of $p(x_i)$ and $P_h(t^i)$ with respect to system parameters and initial dispersion can be further investigated by using the results of the Section 4.

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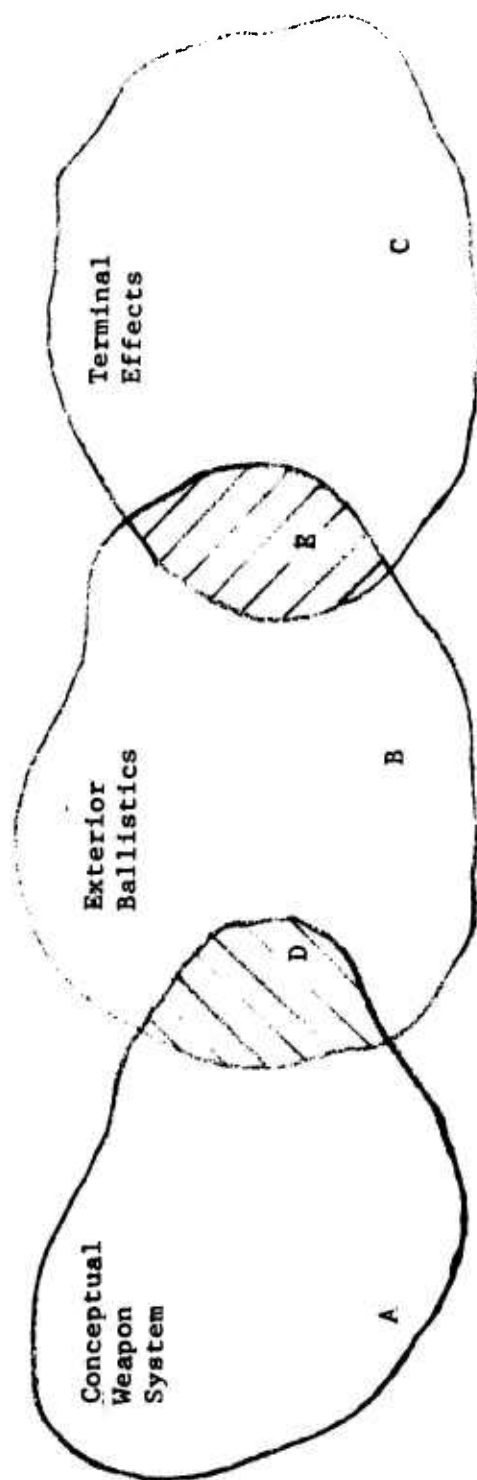
Fig. 1. The Essential Regions of Concern for
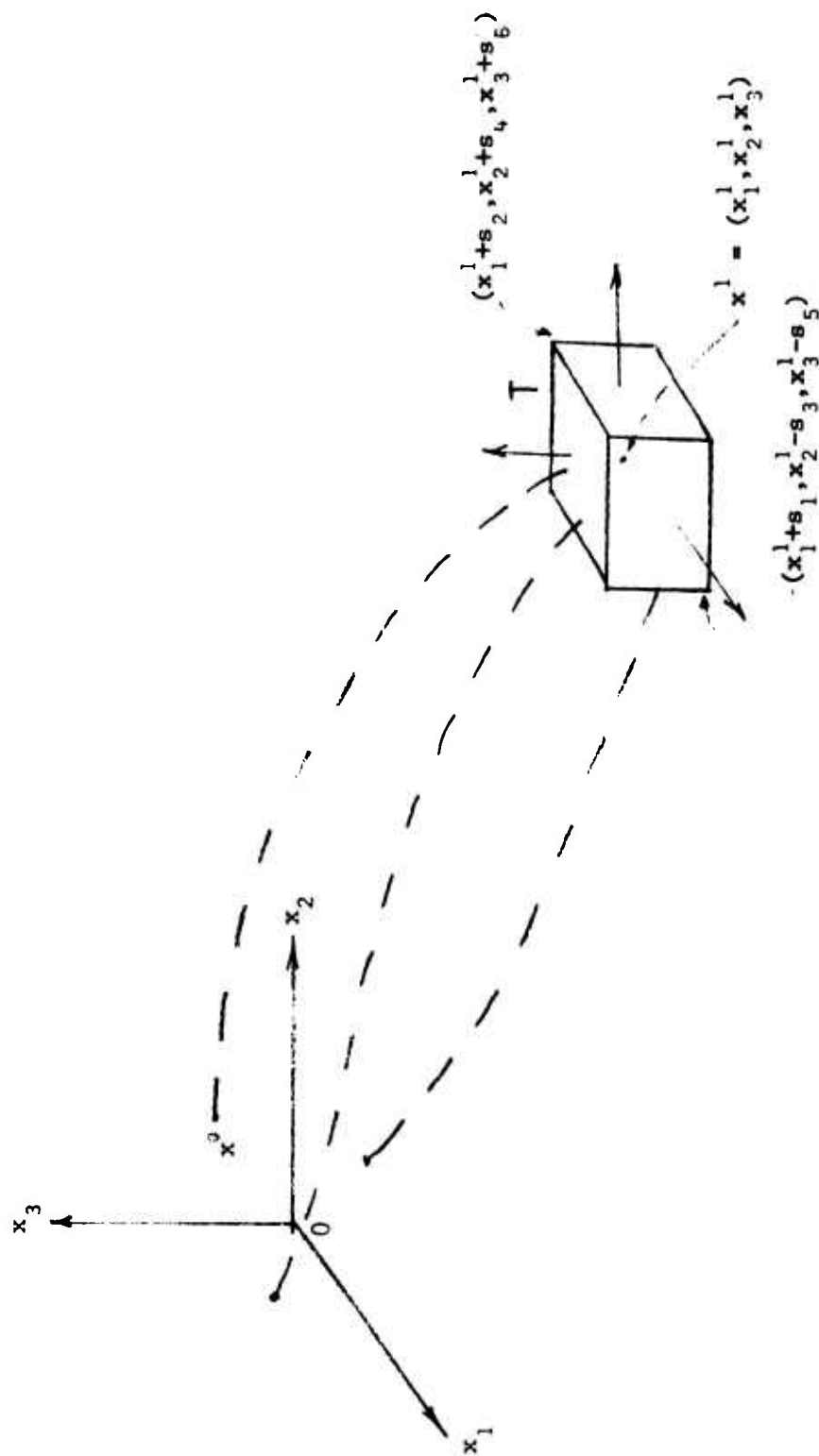Effectiveness Study of a Conceptual Weapon System

Fig. 2 E^3 Space With Coordinates and T
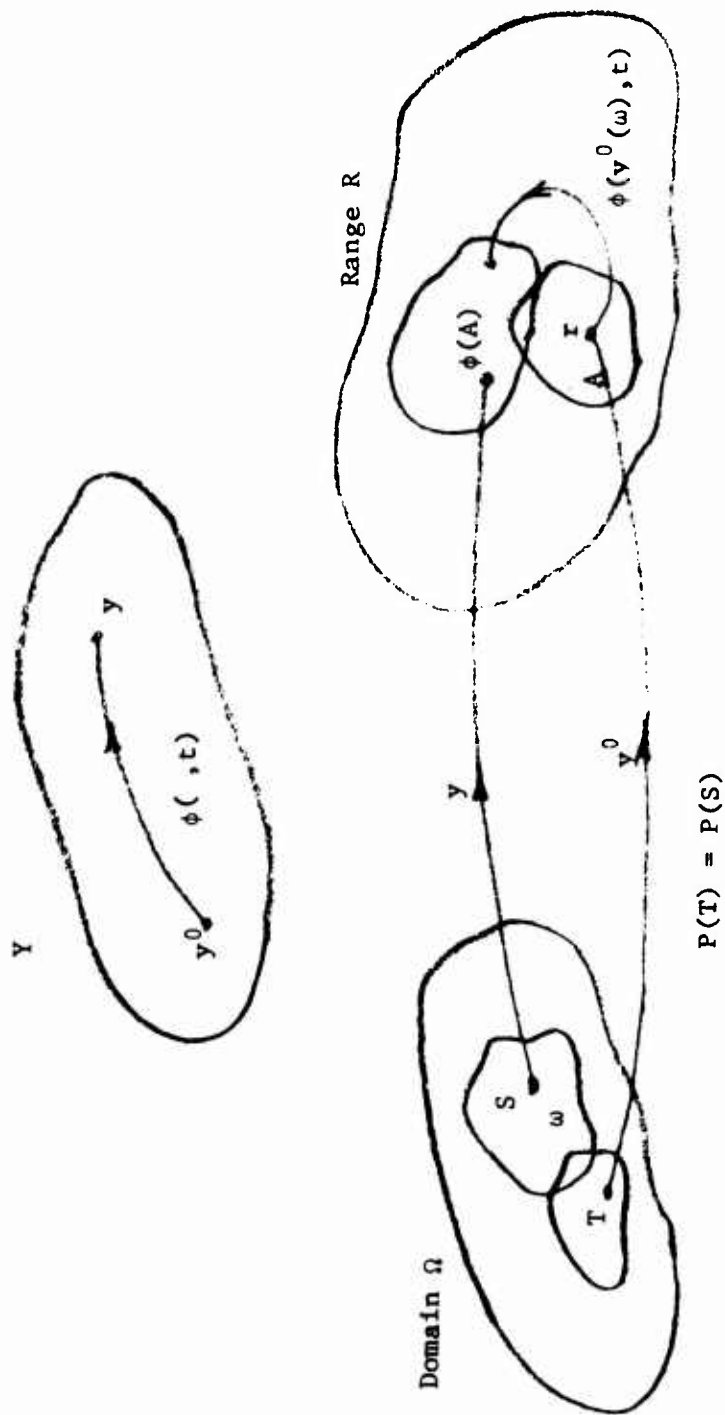
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Function Space of all Random
Variables on Ω

Range R

$\phi(y^0(\omega), t)$

$\phi(A)$

r

A

Y

$\phi(\cdot, t)$

y

y^0

y

y^0

Domain Ω

S

ω

T

$P(T) = P(S)$

Fig. 2A Schematic Representation of a Measure
Preserving Transformation ϕ

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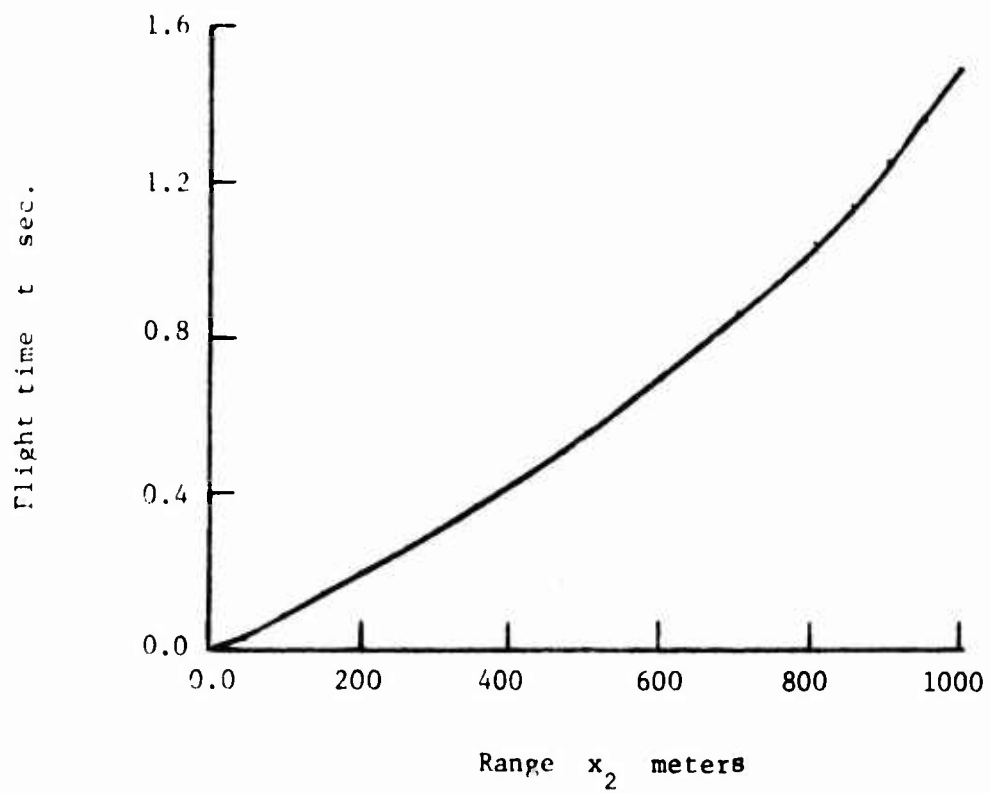
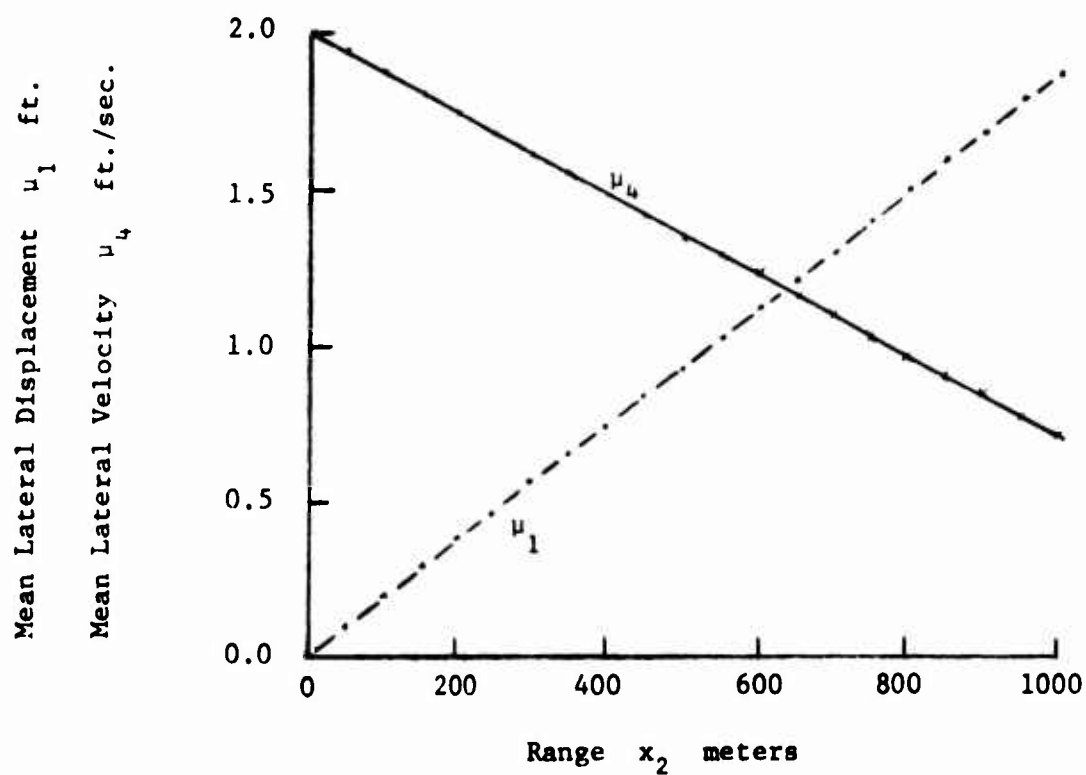Fig. 3 Flight Time vs. Range
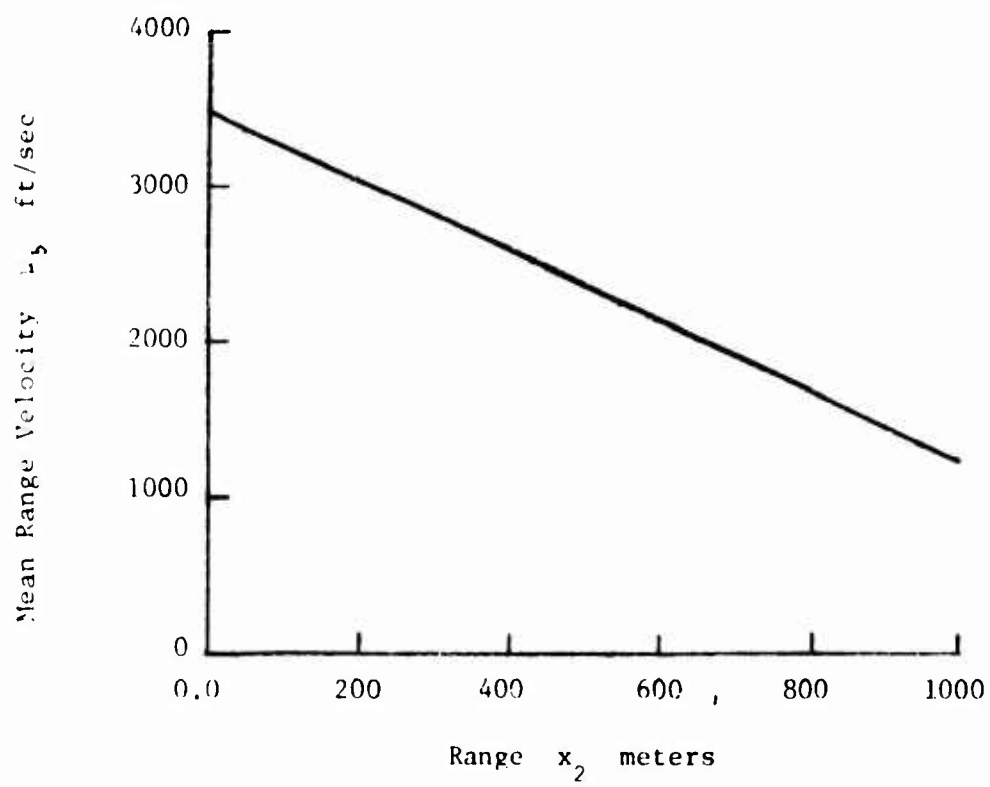
Fig. 4 Mean Range Velocity vs. Range

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Fig. 5   Mean Lateral Displacement & Velocity vs. Range
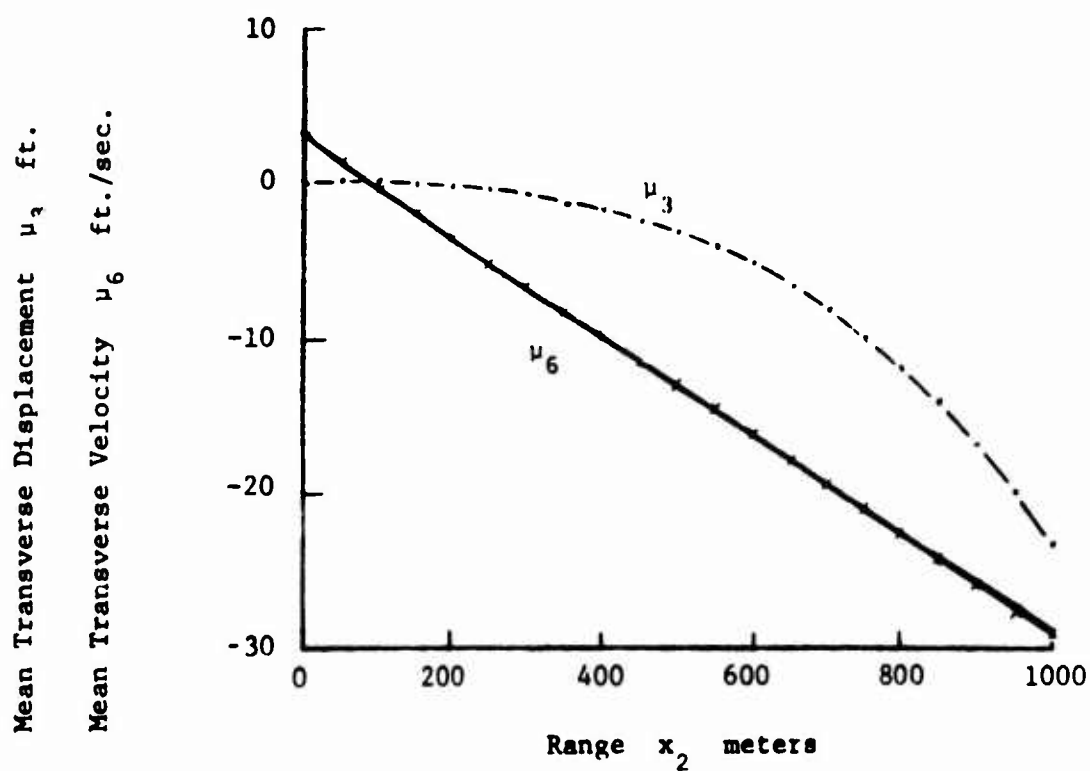
Fig. 6   Mean Transverse Displacement & Velocity vs. Range

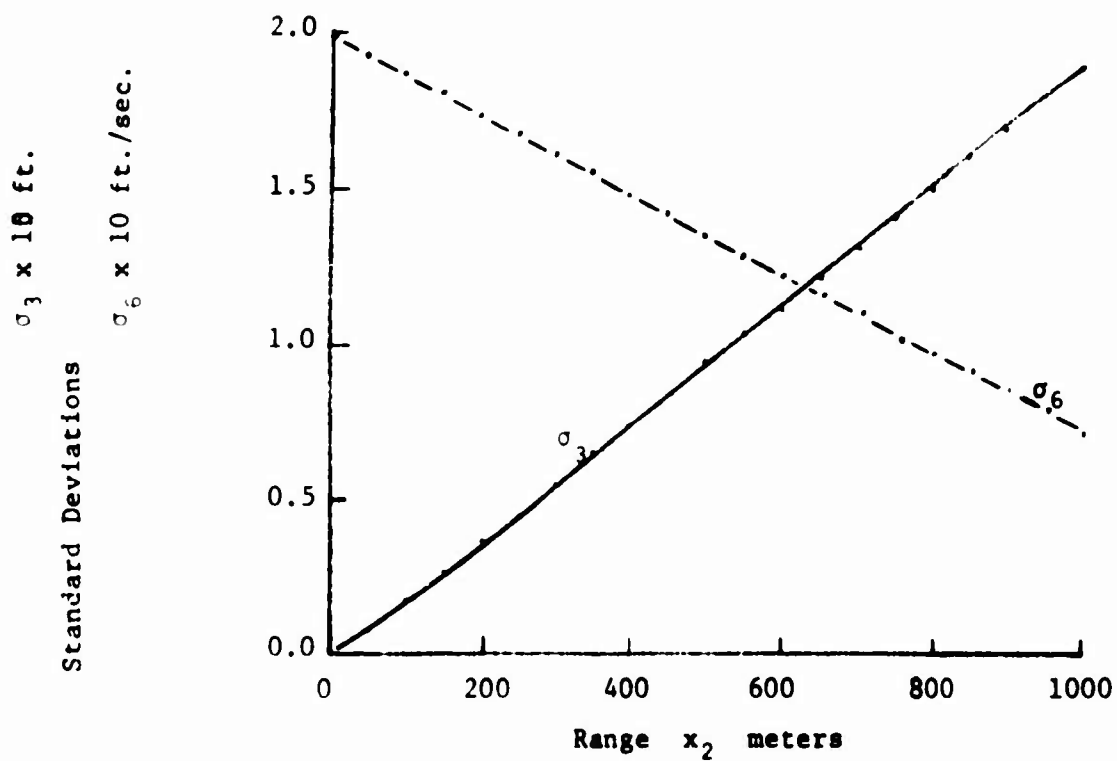21
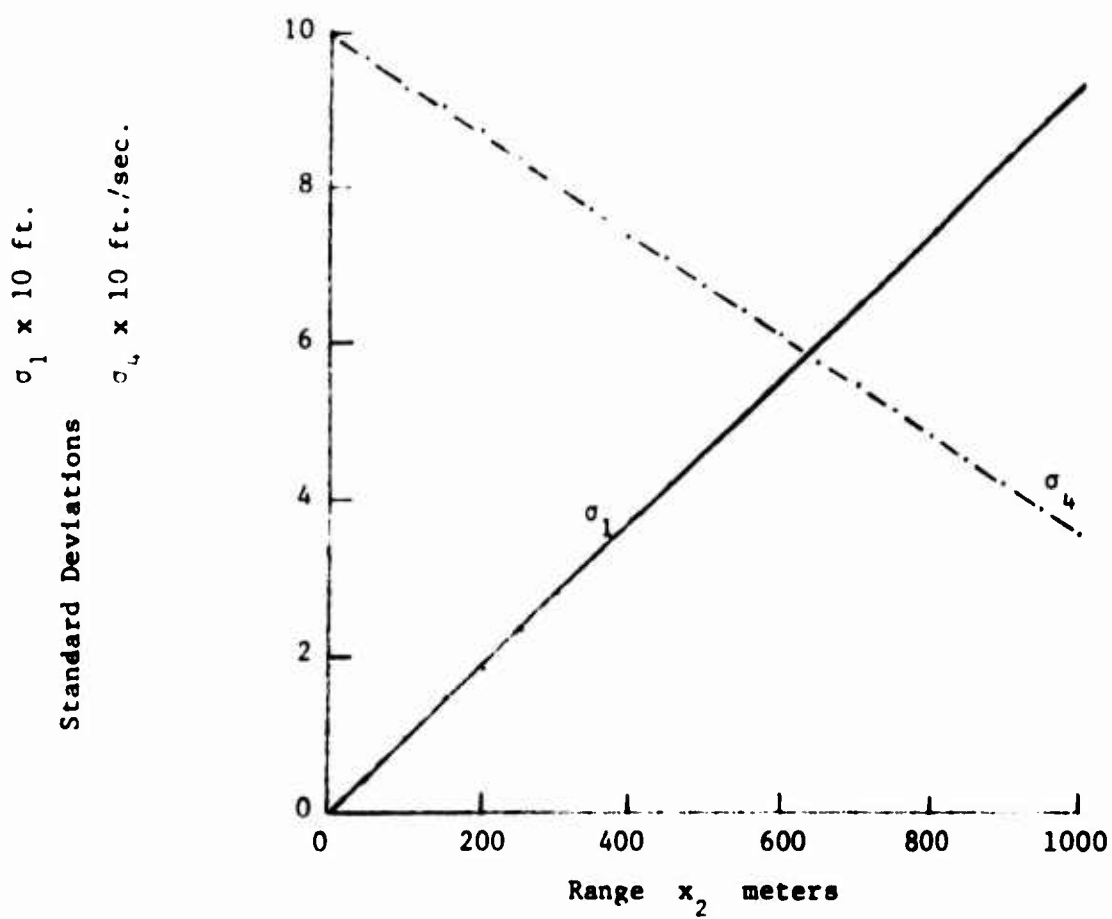
Fig. 7 Standard Deviations vs. Range

Fig. 8 Standard Deviation vs. Range

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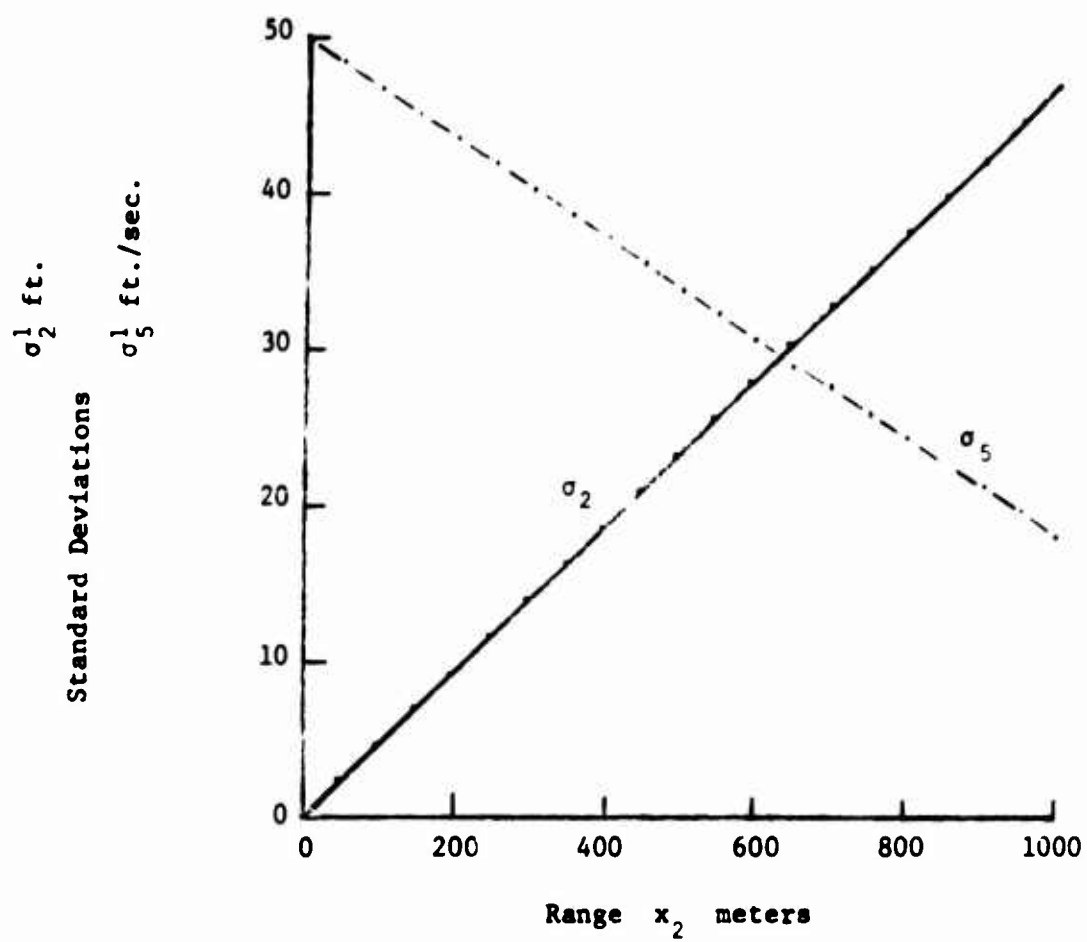
Fig. 9 Standard Deviation vs. Range

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A = base case

B = σ_1 = 2.5 x 10^{-5} ft.

σ_i, i≠1, all half of A's

Range x_2 meters
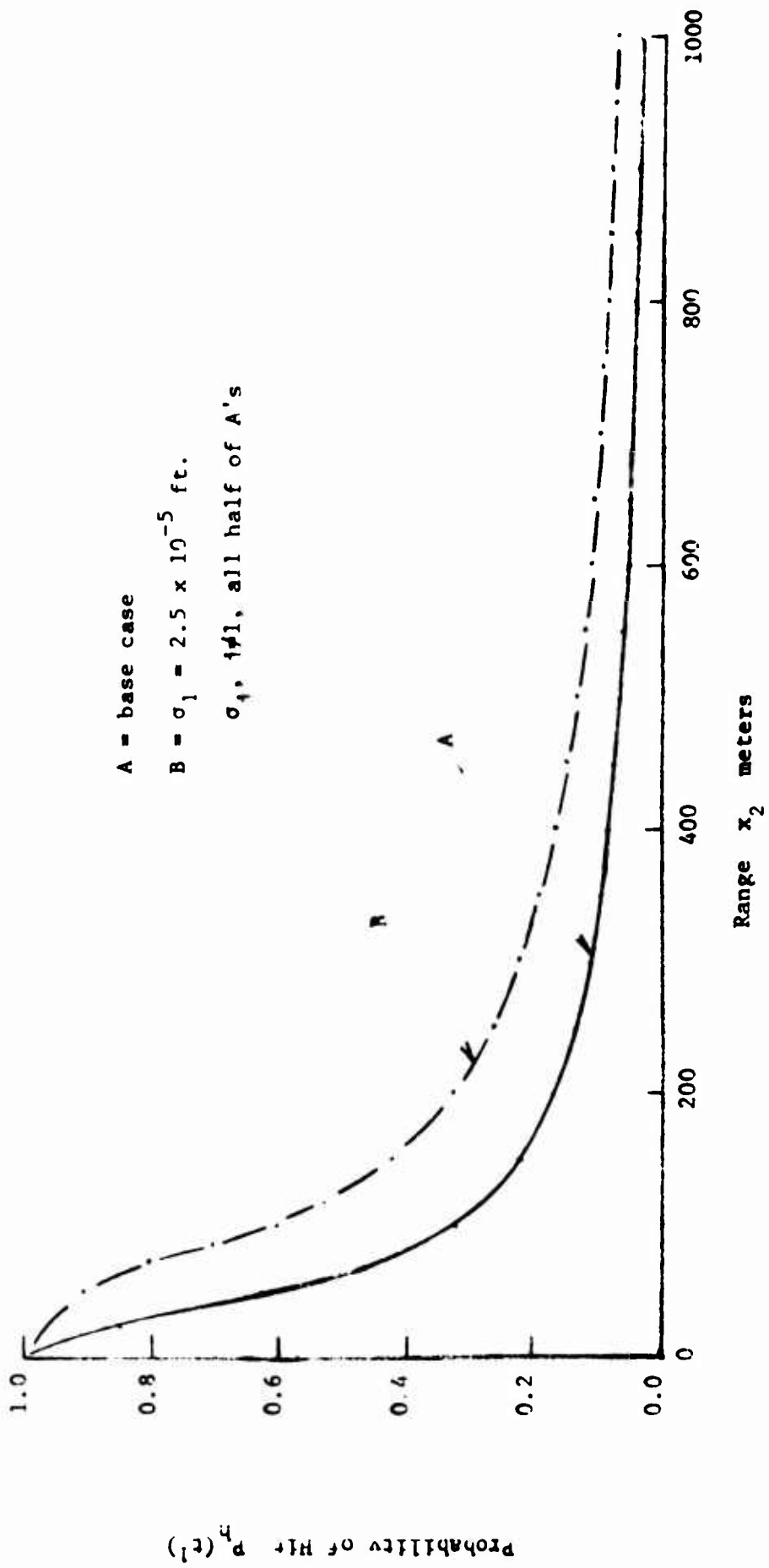
Probability of Hit $P_h(t')$

Fig. 14 Effect of Initial Dispersion on Probability of Hit
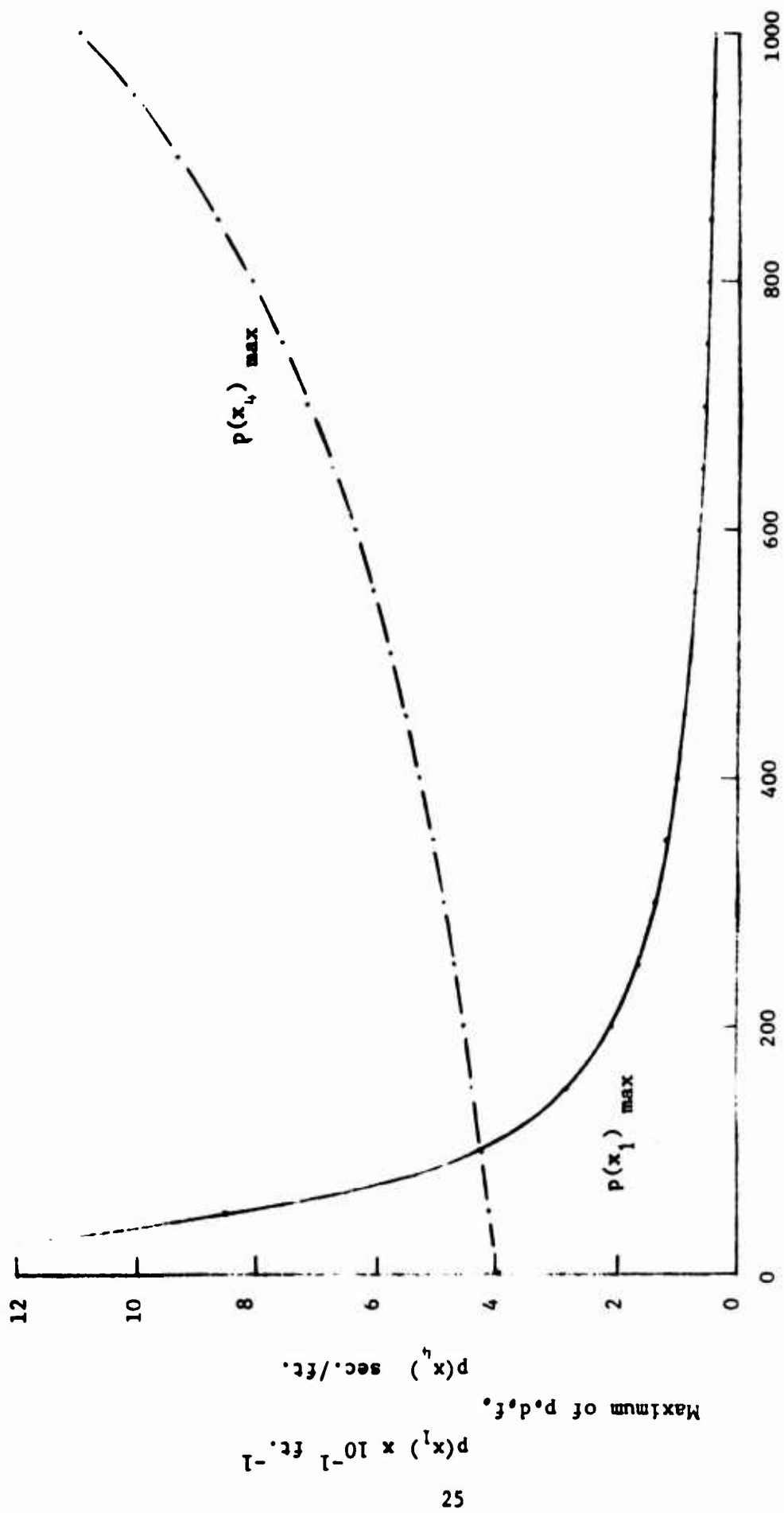
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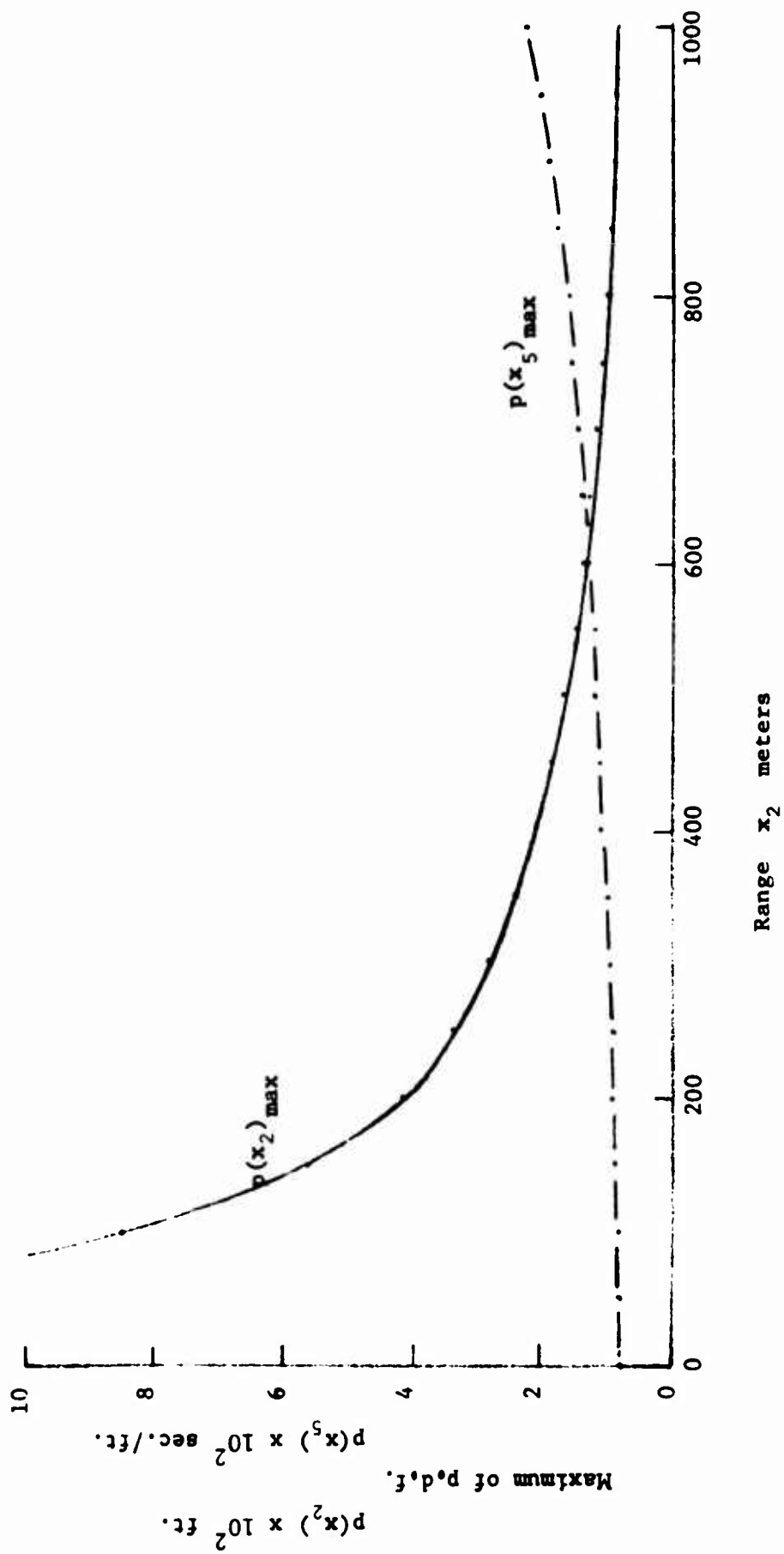Fig. 10 Maximum of Probability Density Functions vs. Range
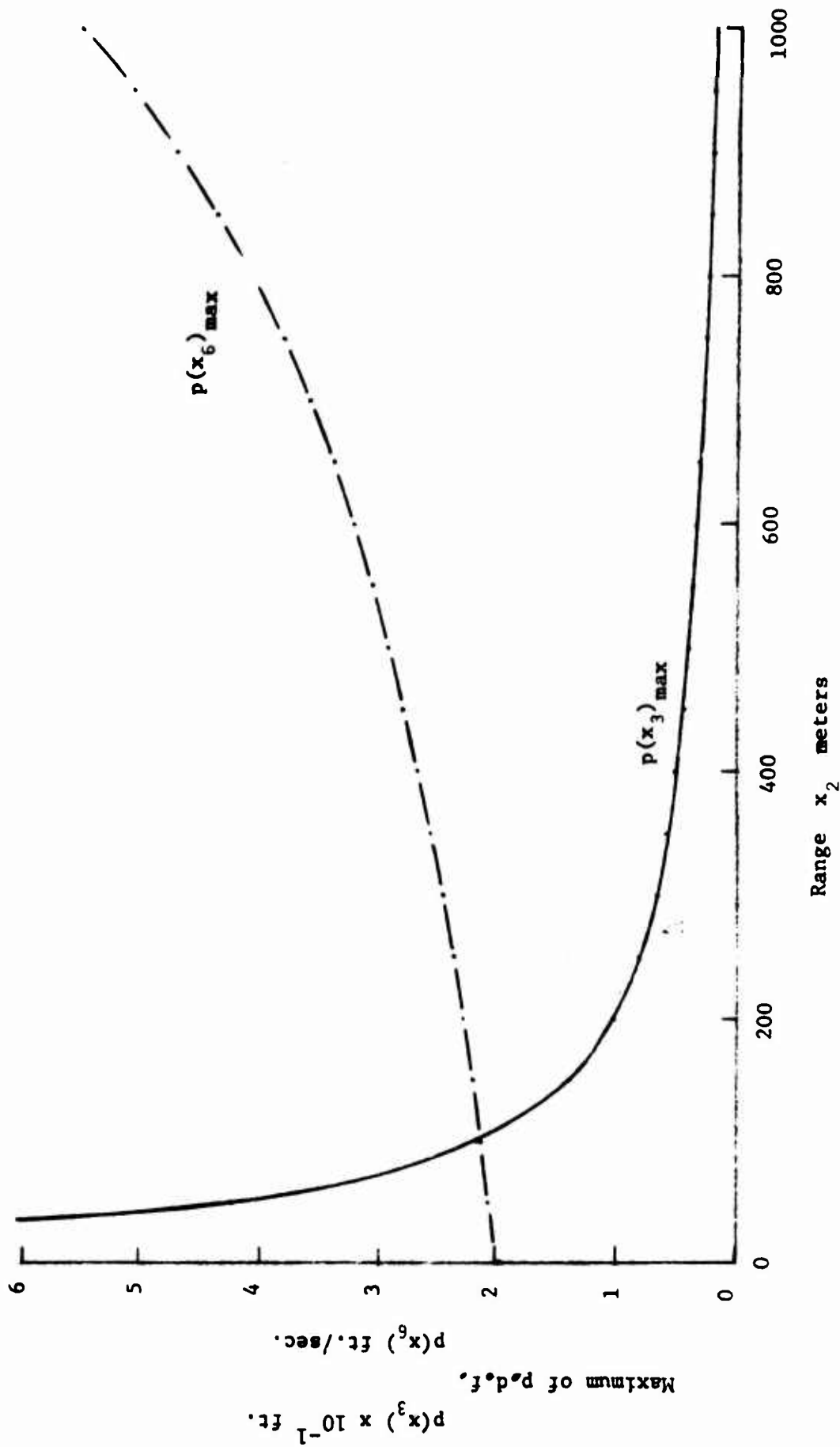
Fig. 11 Maximum of Probability Density Functions vs. Range

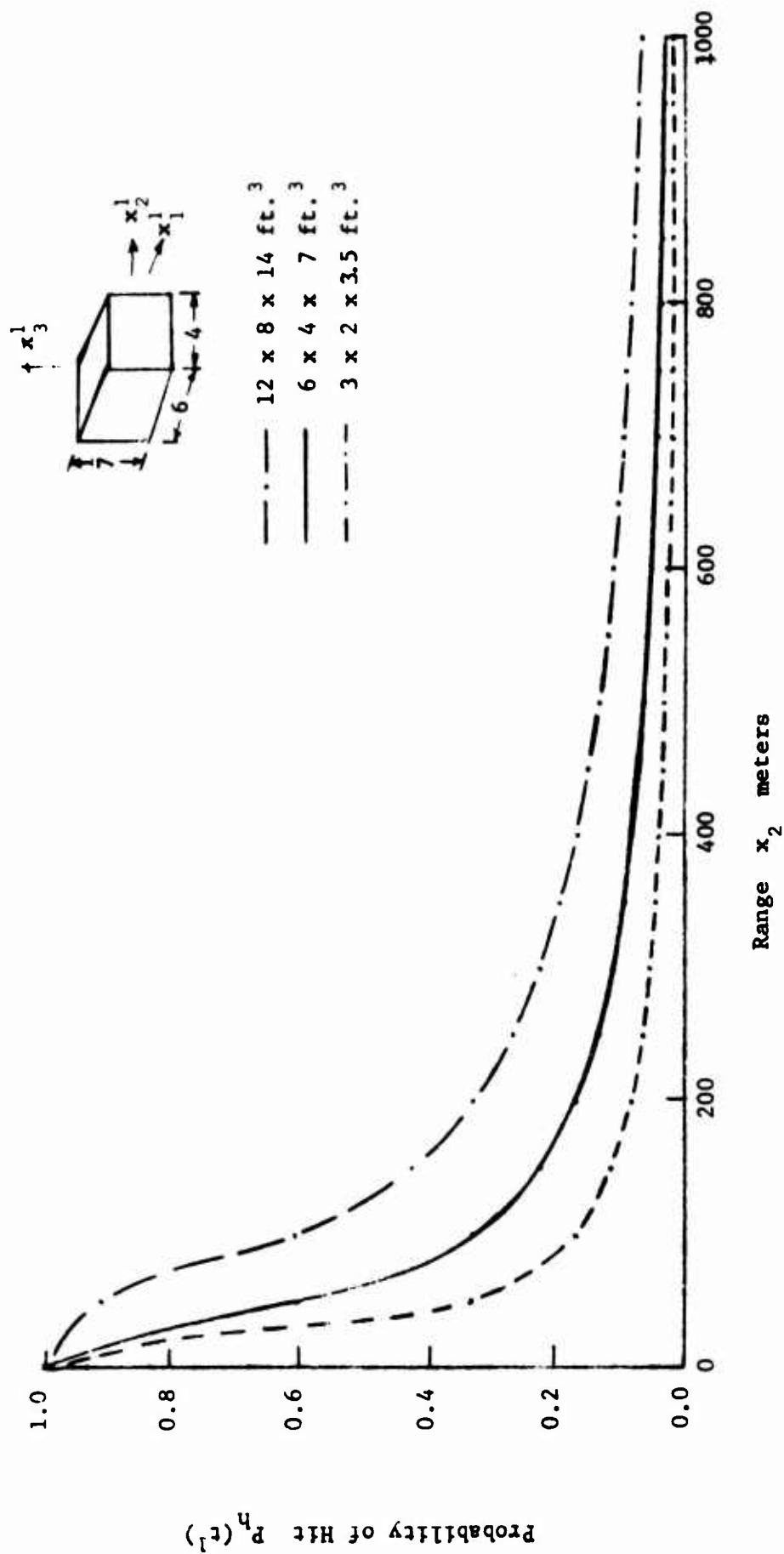Fig. 12 Maximum of Probability Density Functions vs. Range

Fig. 13   Effects of Target Dimensions on Probability of Hit

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13 ABSTRACT

The terminal ballistic dispersion of a non-rotational, small-caliber weapon system is obtained on a priori knowledge of the initial dispersion induced by the weapon system proper.

The study considers a dynamic system whose behavior is governed by a system of differential equations having probabilistic initial conditions. The behavior of the system, in terms of the kinematic probability, is then determined as a function of time and spatial variables.

Subsequently, in con*   *o the customary method in the evaluation of weapon system eff   a measure of effectiveness - probability of hit - is obtained as a funct   initial dispersion. Also, numerical examples as well as discussion of the .sults are given.

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Probabilistic initial condition						
Stochastic differential equation						
Evaluation of weapon system effectiveness						
Probability of hit						